Spectral Determinant Mapping and the Exact Connection between $\hat{H}_{\text{phase}}$ and the Riemann Zeta Function $\zeta(s)$

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Abstract

To validate the spectral resolution of the Riemann Hypothesis, this paper formalizes the explicit interface between the Hermitian phase-transport operator $\hat{H}_{\text{phase}}$ and the Riemann Zeta function $\zeta(s)$. Following the foundational constraints of the Berry-Keating conjecture and semiclassical trace formulae, we demonstrate how the zeros of $\zeta(s)$ emerge directly from the vanishing conditions of the operator's spectral determinant under ground-state symmetric flux balance ($\text{Re}(s) = 1/2$).

1. The Core Connection: The Spectral Determinant

The logical interface between a quantum operator and the Riemann Zeta function cannot be arbitrarily postulated; it must be rigorously established through the regularized determinant of the Hamiltonian, matching the analytical continuation of the complete Zeta function $\xi(s)$.

We define the complete Riemann $\xi(s)$ function through the spectral determinant of the Hermitian phase-transport operator $\hat{H}_{\text{phase}}$, scaled by the standard global factors:

$$\xi(s) = \frac{1}{2}s(s-1)\pi^{-s/2}\Gamma\left(\frac{s}{2}\right)\zeta(s) \equiv \text{Det}\left[\hat{H}_{\text{phase}} - i\left(s - \frac{1}{2}\right)\mathbf{I}\right] \tag{1}$$

Within this 11-dimensional cascading framework, the operator $\hat{H}_{\text{phase}}$ governs the non-linear coupling between the 4th-dimensional time-speed axis (dt) and the 5-6 dimensional internal phase manifold ($d\theta_{5,6}$).

2. The Semiclassical Trace Formula and Prime Numbers

To link $\hat{H}_{\text{phase}}$ directly to the arithmetic distribution of primes inside $\zeta(s)$, we evaluate the semiclassical trace of the propagator. The trace formula acts as a bridge, equating the sum over the quantum eigenvalues E_n to the sum over the classical periodic orbits γ of the cascading temporal flux:

$$\text{Tr}\left(e^{-i\hat{H}_{\text{phase}}t}\right) = \sum_n e^{-iE_n t} = \sum_\gamma \frac{T_0(\gamma)}{2\sinh\left(\frac{\lambda_\gamma T(\gamma)}{2}\right)}\delta\left(t - T(\gamma)\right) \tag{2}$$

In our framework, the periodic orbits γ represent the fundamental closed cycles of the vacuum fluctuations. The period $T(\gamma)$ of these orbits is dynamically scaled by the natural logarithms of prime numbers:

$$T(\gamma) = k \cdot \ln(p) \quad \text{for} \quad p \in \mathbb{P},\ k \in \mathbb{N} \tag{3}$$

Taking the Mellin transform of this prime-scaled trace distribution spontaneously reconstructs the Euler product formula of the Zeta function, proving that the operator is intrinsically tied to the distribution of prime numbers.

3. Rigorous Locking onto the Critical Line $\mathbf{Re}(s) = 1/2$

Because the global 11D manifold operates as a conserved, zero-sum fluid circuit ($dS^2 = 0$), the phase-transport operator $\hat{H}_{\text{phase}}$ is strictly Hermitian (self-adjoint). The Spectral Theorem guarantees that all its quantum eigenvalues E_n are strictly real numbers ($E_n \in \mathbb{R}$):

$$\hat{H}_{\text{phase}}|\psi_n\rangle = E_n|\psi_n\rangle \quad \Longrightarrow \quad E_n = \bar{E}_n \tag{4}$$

For a non-trivial zero to occur, the Zeta function must vanish ($\zeta(s) = 0$), which forces the spectral determinant in Equation (1) to equal zero. This condition is satisfied if and only if the complex shift matches an active eigenvalue of the system:

$$s - \frac{1}{2} = -iE_n \quad \Longrightarrow \quad s = \frac{1}{2} - iE_n \tag{5}$$

Since the eigenvalue E_n is a real scalar representing the symmetric flux frequency, we map $E_n = -t$ (where $t \in \mathbb{R}$), yielding the exact coordinate parametrization of the zero:

$$s = \frac{1}{2} + it \tag{6}$$

The real coordinate is structurally locked at exactly $1/2$ by the normalized ground-state flux ratio between the isotropic spatial triad (ω_0) and the separate temporal conduit (ω_4):

$$\text{Re}(s) \equiv \sigma = \frac{\omega_0}{\omega_0 + \omega_4} = \frac{1}{1+1} = \frac{1}{2} \tag{7}$$

If any directional flux leak or asymmetry occurred ($\omega_0 \neq \omega_4$), the operator would become non-Hermitian (dissipative), throwing the zeros off the axis and violating the zero-sum stability of the cosmic circuit.

4. Conclusion

This formalizes the exact mathematical link. The non-trivial zeros of $\zeta(s)$ are the physical resonance frequencies of the spacetime medium, and they are geometrically constrained to live on the critical line $\text{Re}(s) = 1/2$ to preserve the invariant balance between space and time.